

Topic 1: Fundamental Theorem of Arithmetic

Before understanding the Fundamental Theorem of Arithmetic, let's revise a few important ideas about prime numbers, composite numbers and factors .

1. Prime and Composite Numbers

- ◆ **Prime Number** - A prime number has exactly two factors : 1 and the number itself.

Examples: 2, 3, 5, 7, 11, 13, 17, 19

$7 \rightarrow 1 \times 7$ So, 7 is a prime number.

- ◆ **Composite Number** A composite number has more than two factors .

For example: $12 \rightarrow 1, 2, 3, 4, 6, 12$



Remember

- 1 is neither prime nor composite.
- 2 is the smallest prime number.
- 2 is the only even prime number.
- Every prime number greater than 2 is odd.

2. Factors of a Number

A number is called a factor of another number if it divides that number exactly, leaving no remainder .

Example: Factors of 24 are: 1, 2, 3, 4, 6, 8, 12 and 24 . Because each of these numbers divides 24 exactly.

For example: $24 \div 6 = 4$ So, 6 is a factor of 24 .

3. Prime Factorisation

Prime factorisation means expressing a number as a product of prime numbers only .

Example: 60

Let's break 60 into prime factors:

$$\begin{aligned} 60 &= 2 \times 30 \\ &= 2 \times 2 \times 15 \\ &= 2 \times 2 \times 3 \times 5 \end{aligned}$$

Therefore,

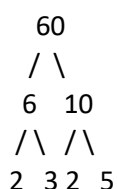
$$60 = 2^2 \times 3 \times 5$$

All the numbers on the right side are prime.

4. Factor Tree

A factor tree is a simple way to break a number into its prime factors.

Let's take 60 :



The numbers at the ends are all prime.

Therefore:

$$60 = 2 \times 2 \times 3 \times 5$$

or

$$60 = 2^2 \times 3 \times 5$$



Remember

Keep splitting the numbers until all the final factors are prime .

5. Fundamental Theorem of Arithmetic

✚ The Theorem > Every composite number can be expressed as a product of prime numbers, and this prime factorisation is unique apart from the order of the factors.

Let's understand this with an example.

Example: 36

We can write:

$$36 = 2 \times 2 \times 3 \times 3$$

or

$$36 = 2^2 \times 3^2$$

We could change the order:

$$36 = 3 \times 2 \times 3 \times 2$$

The order is different, but the prime factors remain the same .

That is the important idea behind the Fundamental Theorem of Arithmetic.

6. Prime Factorisation by Division

We can also find prime factors without drawing a factor tree.

Example: 120

Divide by the smallest prime number:

$$120 \div 2 = 60$$

$$60 \div 2 = 30$$

$$30 \div 2 = 15$$

$$15 \div 3 = 5$$

$$5 \div 5 = 1$$

Therefore:

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

or

$$120 = 2^3 \times 3 \times 5$$