Topic 1: Fundamental Theorem of Arithmetic

Before understanding the Fundamental Theorem of Arithmetic, let's revise a few important ideas about prime numbers, composite numbers and factors .

1. Prime and Composite Numbers

- **Prime Number** - A prime number has exactly two factors : 1 and the number itself.
  Examples: 2, 3, 5, 7, 11, 13, 17, 19

  7 → 1 × 7 So, 7 is a prime number.
- **Composite Number**A composite number has more than two factors .
  For example: 12 → 1, 2, 3, 4, 6, 12

> 💡 **Remember**
> - 1 is neither prime nor composite.
> - 2 is the smallest prime number.
> - 2 is the only even prime number.
> - Every prime number greater than 2 is odd.

2. Factors of a Number

A number is called a factor of another number if it divides that number exactly, leaving no remainder .

Example: Factors of 24 are: 1, 2, 3, 4, 6, 8, 12 and 24 . Because each of these numbers divides 24 exactly.

For example: 24 ÷ 6 = 4 So, 6 is a factor of 24 .

3. Prime Factorisation

Prime factorisation means expressing a number as a product of prime numbers only .

Example: 60

Let's break 60 into prime factors:

60 = 2 × 30

= 2 × 2 × 15

= 2 × 2 × 3 × 5

Therefore,

$60 = 2^2 \times 3 \times 5$

All the numbers on the right side are prime.

4. Factor Tree 🌳

A factor tree is a simple way to break a number into its prime factors.

Let's take 60 :

```
      60
     / \
    6   10
   / \  / \
  2  3 2  5
```

The numbers at the ends are all prime.

Therefore:

$60 = 2 \times 2 \times 3 \times 5$

or

$60 = 2^2 \times 3 \times 5$

> 💡 Remember
>
> Keep splitting the numbers until all the final factors are prime .

5. Fundamental Theorem of Arithmetic

📌 The Theorem > Every composite number can be expressed as a product of prime numbers, and this prime factorisation is unique apart from the order of the factors.

Let's understand this with an example.

Example: 36

We can write:

$36 = 2 \times 2 \times 3 \times 3$

or

$36 = 2^2 \times 3^2$

We could change the order:

$36 = 3 \times 2 \times 3 \times 2$

The order is different, but the prime factors remain the same .

That is the important idea behind the Fundamental Theorem of Arithmetic.

6. Prime Factorisation by Division

We can also find prime factors without drawing a factor tree.

Example: 120

Divide by the smallest prime number:

$120 \div 2 = 60$

$60 \div 2 = 30$

$30 \div 2 = 15$

$15 \div 3 = 5$

$5 \div 5 = 1$

Therefore:

$120 = 2 \times 2 \times 2 \times 3 \times 5$

or

$120 = 2^3 \times 3 \times 5$